

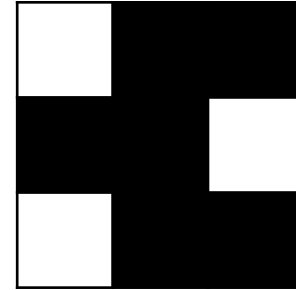
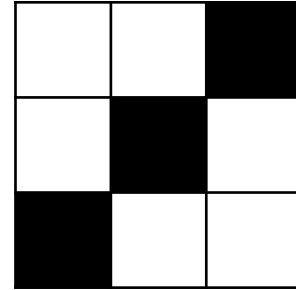
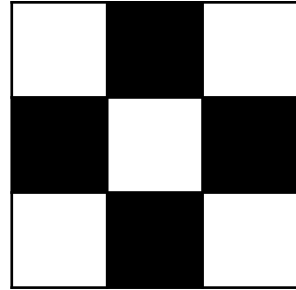
CSE 417T: Introduction to Machine Learning

Lecture 2: Generalization

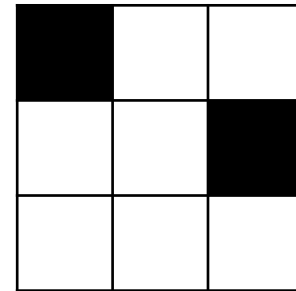
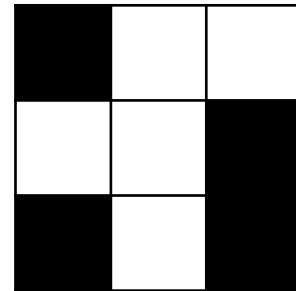
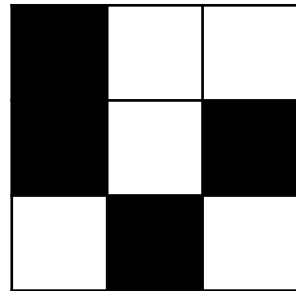
Henry Chai

08/30/18

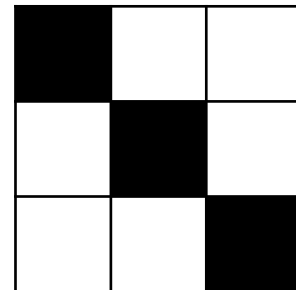
Puzzle



$$f(x) = +1$$

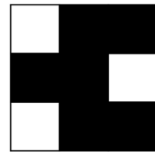
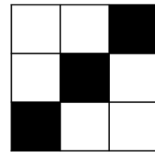
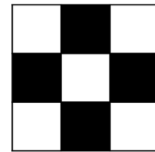


$$f(x) = -1$$

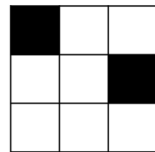
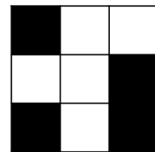
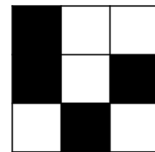


$$f(x) = ???$$

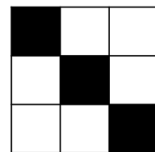
An Answer



$f(x) = +1$



$f(x) = -1$



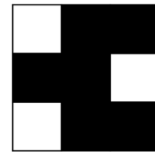
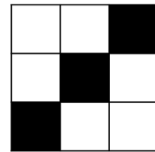
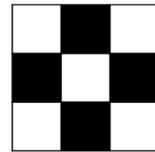
$f(x) = ???$

$$h(x) = \begin{cases} +1 & \text{if symmetric} \\ -1 & \text{otherwise} \end{cases}$$

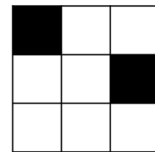
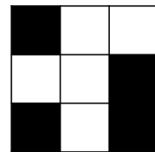
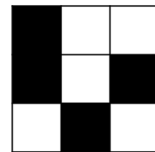


$$h\left(\begin{array}{|c|c|c|} \hline \blacksquare & \square & \square \\ \hline \square & \blacksquare & \square \\ \hline \square & \square & \blacksquare \\ \hline \end{array}\right) = +1$$

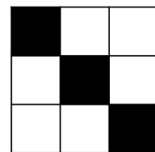
An Answer



$f(x) = +1$



$f(x) = -1$



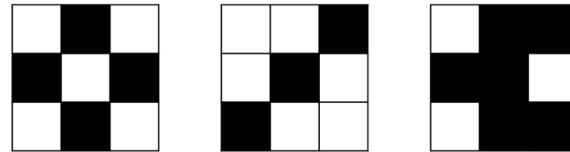
$f(x) = ???$

$$h(x) = \begin{cases} +1 & \text{if top left is white} \\ -1 & \text{otherwise} \end{cases}$$

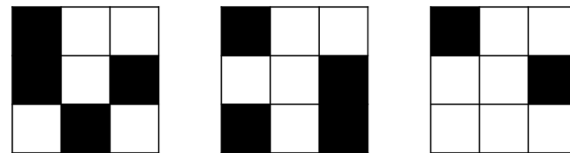


$$h\left(\begin{array}{|c|c|c|} \hline \text{black} & \text{white} & \text{white} \\ \hline \text{white} & \text{black} & \text{white} \\ \hline \text{white} & \text{white} & \text{black} \\ \hline \end{array}\right) = -1$$

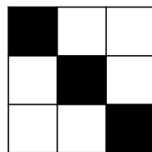
An Answer



$f(x) = +1$



$f(x) = -1$



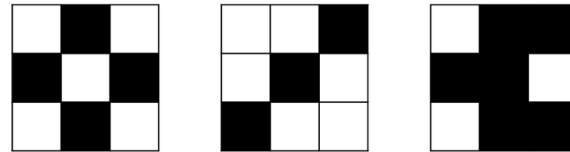
$f(x) = ???$

$$h(x) = \begin{cases} +1 & \text{if } x \in \{ \begin{smallmatrix} \blacksquare & \square & \square \\ \square & \blacksquare & \square \\ \square & \square & \blacksquare \end{smallmatrix}, \begin{smallmatrix} \square & \square & \blacksquare \\ \square & \blacksquare & \square \\ \blacksquare & \square & \square \end{smallmatrix}, \begin{smallmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \square & \square \\ \square & \blacksquare & \square \end{smallmatrix} \} \\ -1 & \text{otherwise} \end{cases}$$

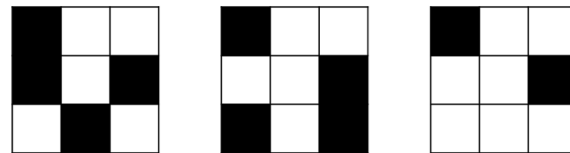


$$h\left(\begin{array}{|c|c|c|} \hline \blacksquare & \square & \square \\ \hline \square & \blacksquare & \square \\ \hline \square & \square & \blacksquare \\ \hline \end{array}\right) = -1$$

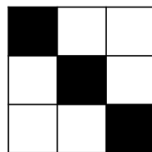
An Answer



$f(x) = +1$



$f(x) = -1$



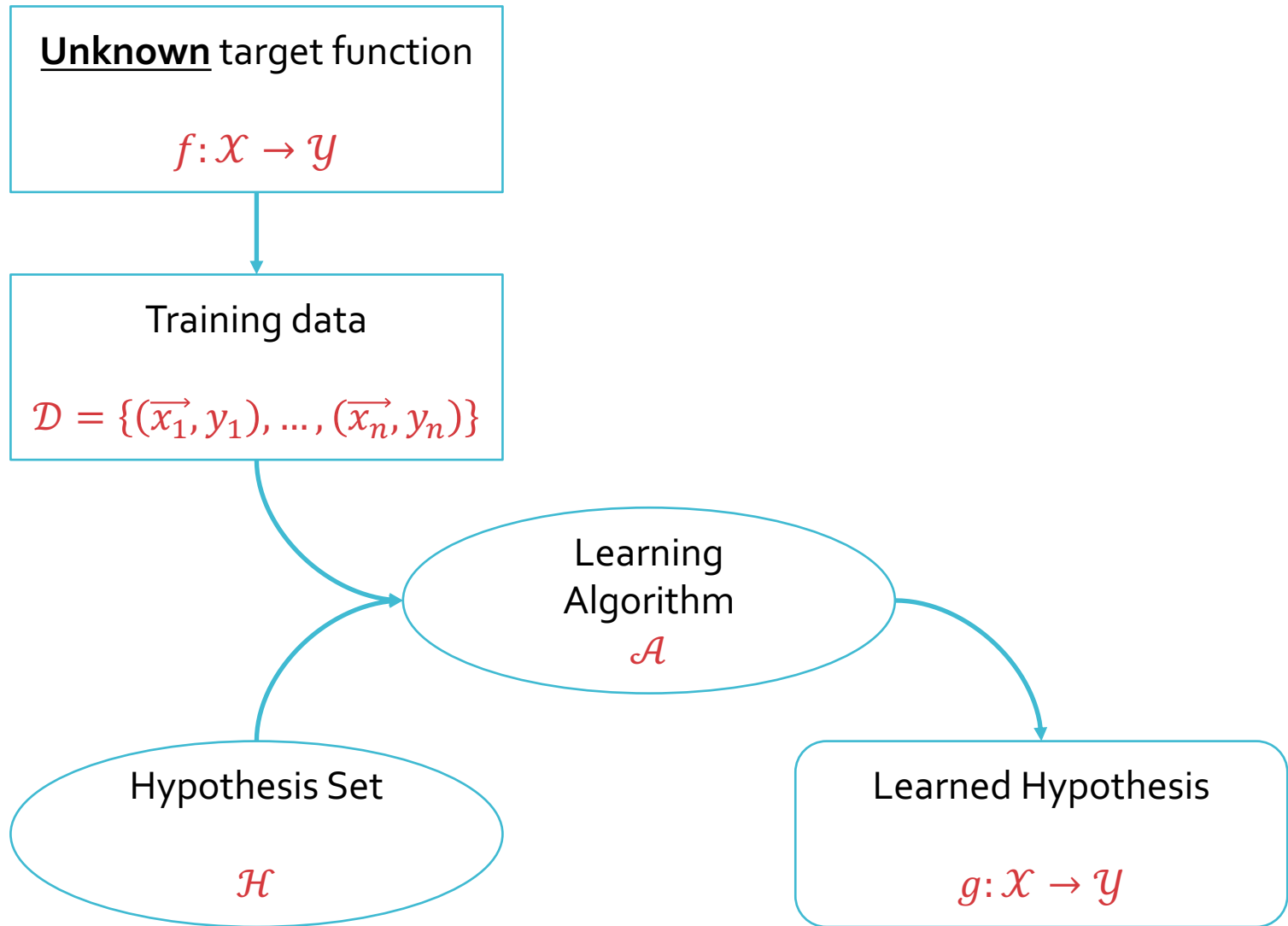
$f(x) = ???$

$$h(x) = \begin{cases} +1 & \text{if } x \in \{ \begin{smallmatrix} \blacksquare & \square & \square \\ \square & \blacksquare & \square \\ \square & \square & \blacksquare \end{smallmatrix} \} \\ -1 & \text{otherwise} \end{cases}$$

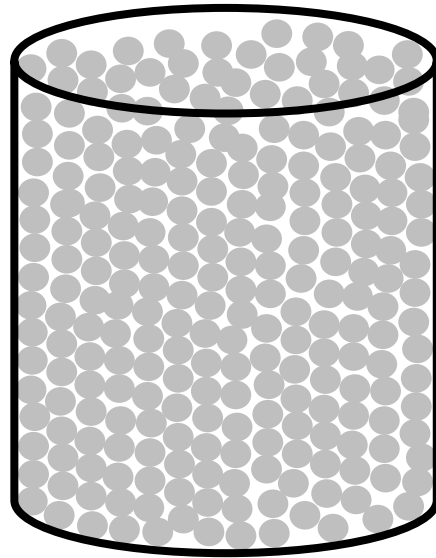


$$h \left(\begin{smallmatrix} \blacksquare & \square & \square \\ \square & \blacksquare & \square \\ \square & \square & \blacksquare \end{smallmatrix} \right) = +1$$

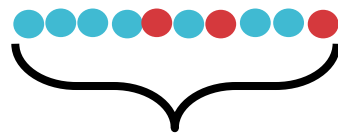
Recall



Analogy



μ = fraction of red marbles in bin



n i. i. d. samples

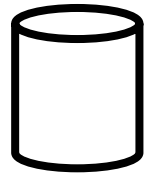
v = fraction of red marbles in sample

Does v say anything about μ ?

Hoeffding's Inequality

- μ = fraction of red marbles in bin
- ν = fraction of red marbles in a sample of size n
- $P\{|\nu - \mu| > \epsilon\} \leq 2e^{-2\epsilon^2 n}$
- As n increases, RHS decreases
- As ϵ decreases, RHS increases

Connection to Learning



= input space (\mathcal{X})

● = point in the input space (\vec{x})

●●●●●●●● = training data (\mathcal{D})

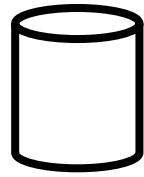
● = a point classified correctly by a specified hypothesis h

● = a point classified incorrectly by a specified hypothesis h

v = fraction of training data classified incorrectly by h

μ = fraction of points in the input space classified incorrectly by h

Connection to Learning



= input space (\mathcal{X})

● = point in the input space (\vec{x})

●●●●●●●● = training data (\mathcal{D})

● = a point classified correctly by a specified hypothesis h

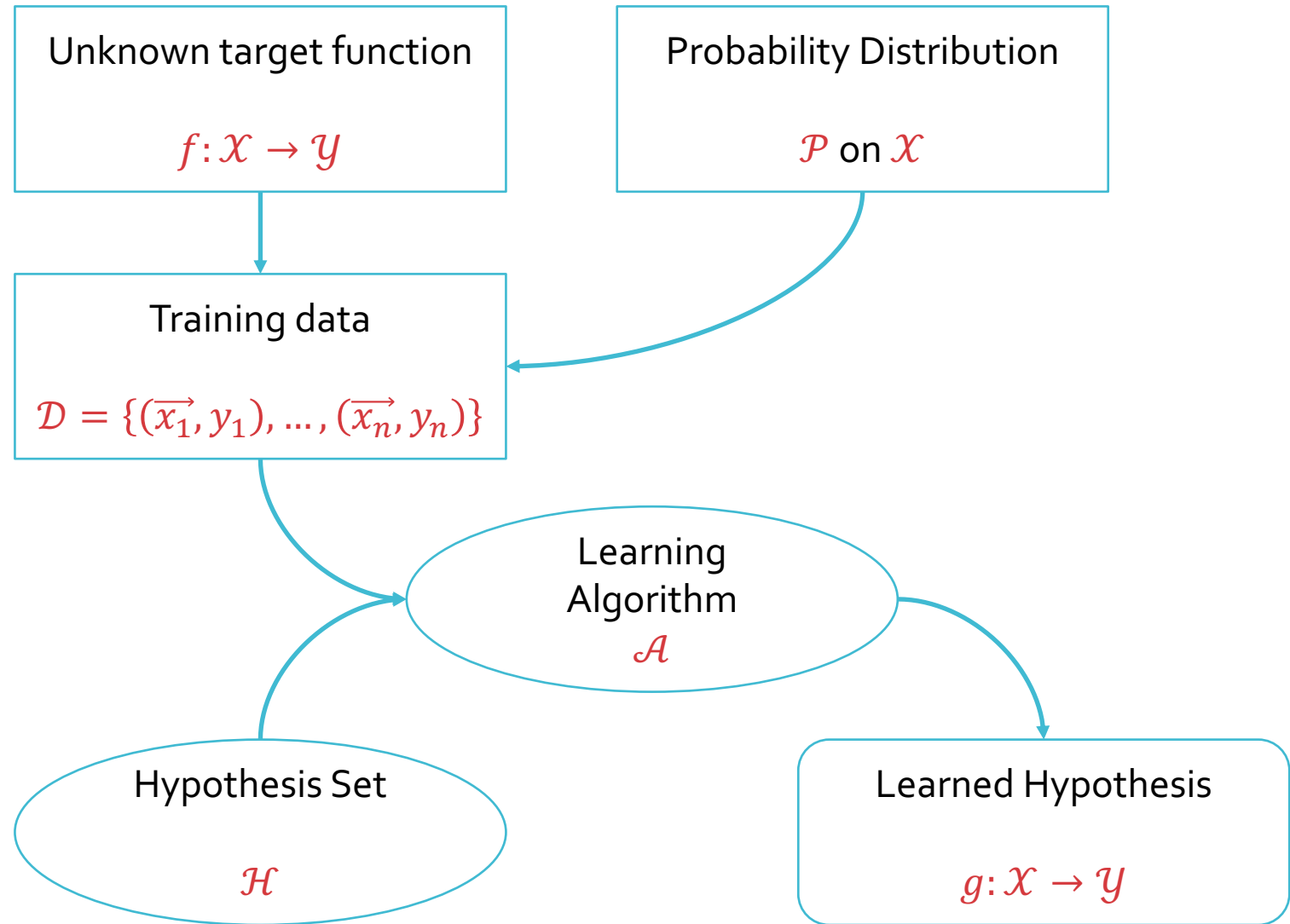
● = a point classified incorrectly by a specified hypothesis h

ν = fraction of training data classified incorrectly by h ($E_{in}(h)$)

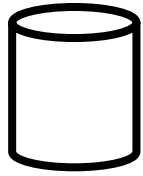
μ = fraction of points in the input space classified incorrectly by h ($E_{out}(h)$)

$$P\{|E_{in}(h) - E_{out}(h)| > \epsilon\} \leq 2e^{-2\epsilon^2 n}$$

Formal Setup



Connection to Validation Learning



= input space (\mathcal{X})



= point in the input space (\vec{x})



= training data (\mathcal{D})



= a point classified correctly **by a specified hypothesis h**



= a point classified incorrectly **by a specified hypothesis h**

v

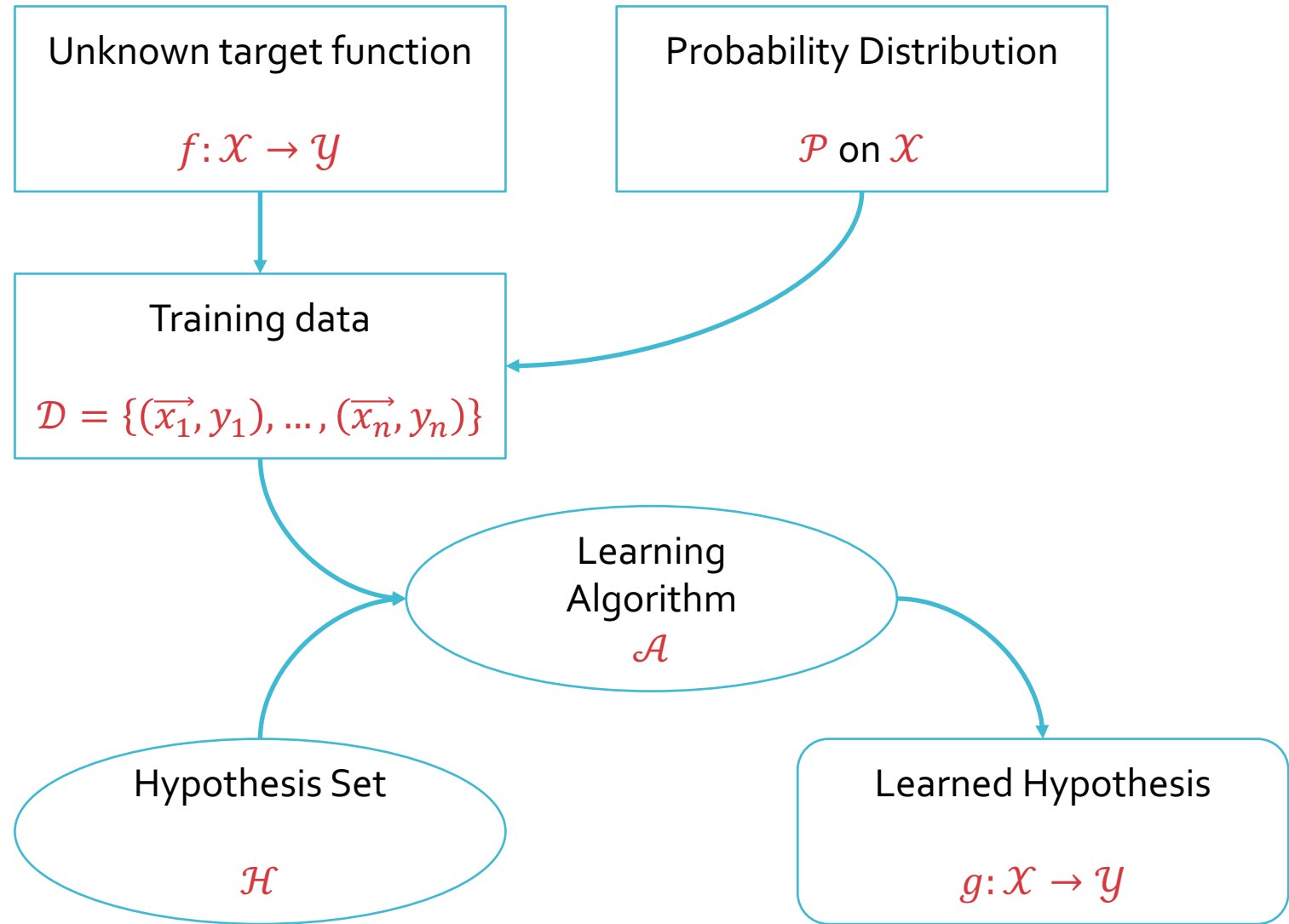
= fraction of training data classified incorrectly **by h** ($E_{in}(h)$)

μ

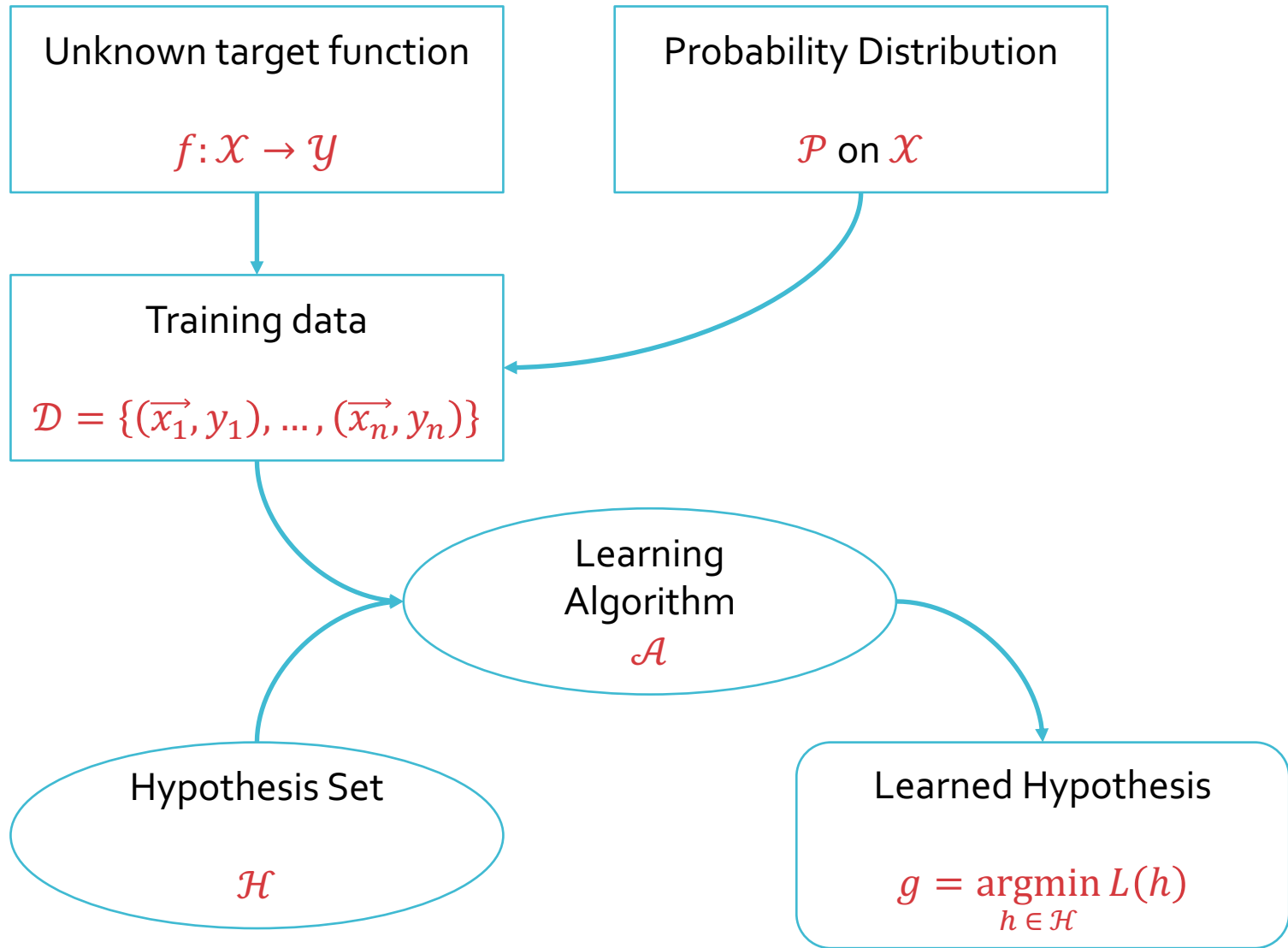
= fraction of all possible data classified incorrectly **by h** ($E_{out}(h)$)

$$P\{|E_{in}(h) - E_{out}(h)| > \epsilon\} \leq 2e^{-2\epsilon^2 n}$$

Formal Setup



Formal Setup



Another Analogy

- If you toss a fair coin 20 times, the probability that it comes up heads 20 times is $2^{-20} \approx 1e-6$
- If you toss 2^{20} fair coins 20 times each, the probability that at least one coin comes up heads 20 times is...
... $\approx 1 - \frac{1}{e} \approx 0.63$

Another Analogy

- Take any coin that came up all heads and apply Hoeffding's inequality
- $P\{|E_{in}(g) - E_{out}(g)| > \epsilon\} \leq 2e^{-2\epsilon^2 n}$
- $P\{|\text{Fraction of tails in 20 trials} - \text{Fraction of tails in } \infty \text{ trials}| > \epsilon\} \leq 2e^{-40\epsilon^2}$
- $P\{|0 - P\{\text{this coin coming up tails}\}| > \epsilon\} \leq 2e^{-40\epsilon^2}$
- $P\left\{P\{\text{this coin coming up tails}\} > \frac{1}{4}\right\} \leq 2e^{-2.5}$

Hoeffding's Inequality (Corrected)

- Suppose \mathcal{H} is finite i.e. $\mathcal{H} = \{h_1, \dots, h_m\}$

$$P\{|E_{in}(g) - E_{out}(g)| > \epsilon\}$$

$$\leq P\left\{\bigcup_{j=1}^m |E_{in}(h_j) - E_{out}(h_j)| > \epsilon\right\}$$

$$\leq \sum_{j=1}^m P\{|E_{in}(h_j) - E_{out}(h_j)| > \epsilon\}$$

$$\leq \sum_{j=1}^m 2e^{-2\epsilon^2 n}$$

Hoeffding's Inequality (Corrected)

- Suppose \mathcal{H} is finite i.e. $\mathcal{H} = \{h_1, \dots, h_m\}$
- $E_{in}(g)$ = in-sample error of best hypothesis in \mathcal{H}
- $E_{out}(g)$ = out-of-sample error of best hypothesis in \mathcal{H}
- $P\{|E_{in}(g) - E_{out}(g)| > \epsilon\} \leq 2(m)e^{-2\epsilon^2 n}$
- As n increases, RHS **decreases**
- As ϵ decreases, RHS **increases**
- As m increases, RHS **increases**