# CSE 417T: Introduction to Machine Learning

# Lecture 3: Error and Noise

Henry Chai

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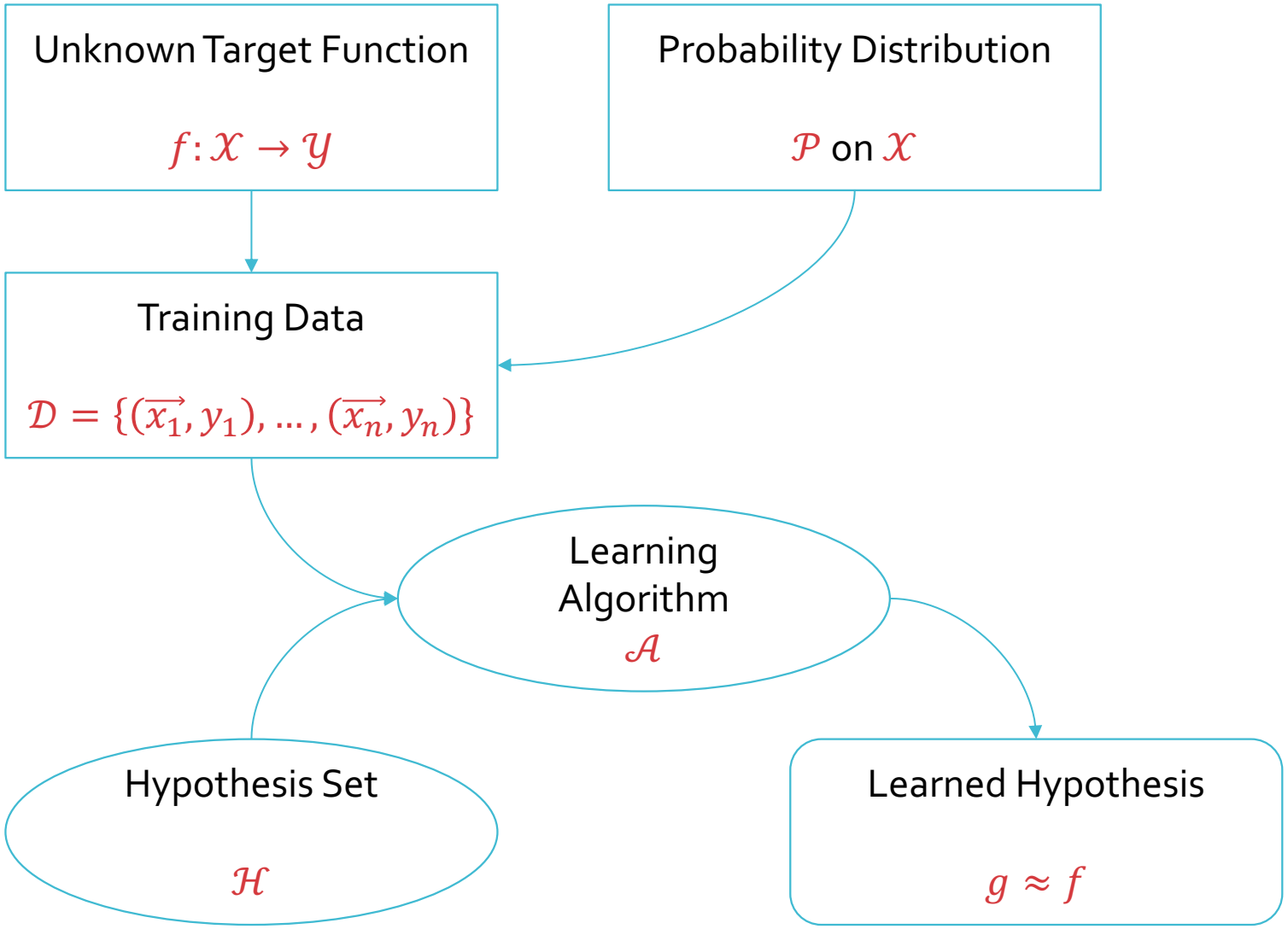
# Recall


Unknown Target Function
$f: \mathcal{X} \rightarrow \mathcal{Y}$
Probability Distribution
$\mathcal{P}$ on $\mathcal{X}$
Training Data
$\mathcal{D} = \{(\overrightarrow{x_1}, y_1), \ldots, (\overrightarrow{x_n}, y_n)\}$
Learning Algorithm
$\mathcal{A}$
Hypothesis Set
$\mathcal{H}$
Learned Hypothesis
$g \approx f$

# Error: is $h \approx f$ ?

- $E(h, f)$ is a function that measures how close $h$ is to $f$: the smaller the better
- $g = \underset{h \in \mathcal{H}}{\operatorname{argmin}} E(h, f)$
- $E$ is usually defined in terms of a pointwise error function $e(h, f, \vec{x})$
  - Binary error (classification): $e(h, f, \vec{x}) = [\![ f(\vec{x}) \neq h(\vec{x}) ]\!]$
  - Squared error (regression): $e(h, f, \vec{x}) = \left( f(\vec{x}) - h(\vec{x}) \right)^2$

# Error: is $h \approx f$ ?

- In-sample error: $E_{in}(h) = \frac{1}{n}\sum_{i=1}^{n} e(h, f, \overrightarrow{x_i})$
- Out-of-sample error: $E_{out}(h) = \mathbb{E}_{x \sim \mathcal{P}}[e(h, f, \vec{x})]$

| | | $f(\vec{x})$ | |
|---|---|---|---|
| | | +1 | -1 |
| $h(\vec{x})$ | +1 | No error | False positive |
| | -1 | False negative | No error |

# Error: Classification

# Error: Classification

- Fingerprint recognition:
  - Inputs are fingerprints
  - Outputs: +1 means "you", -1 means "not you"

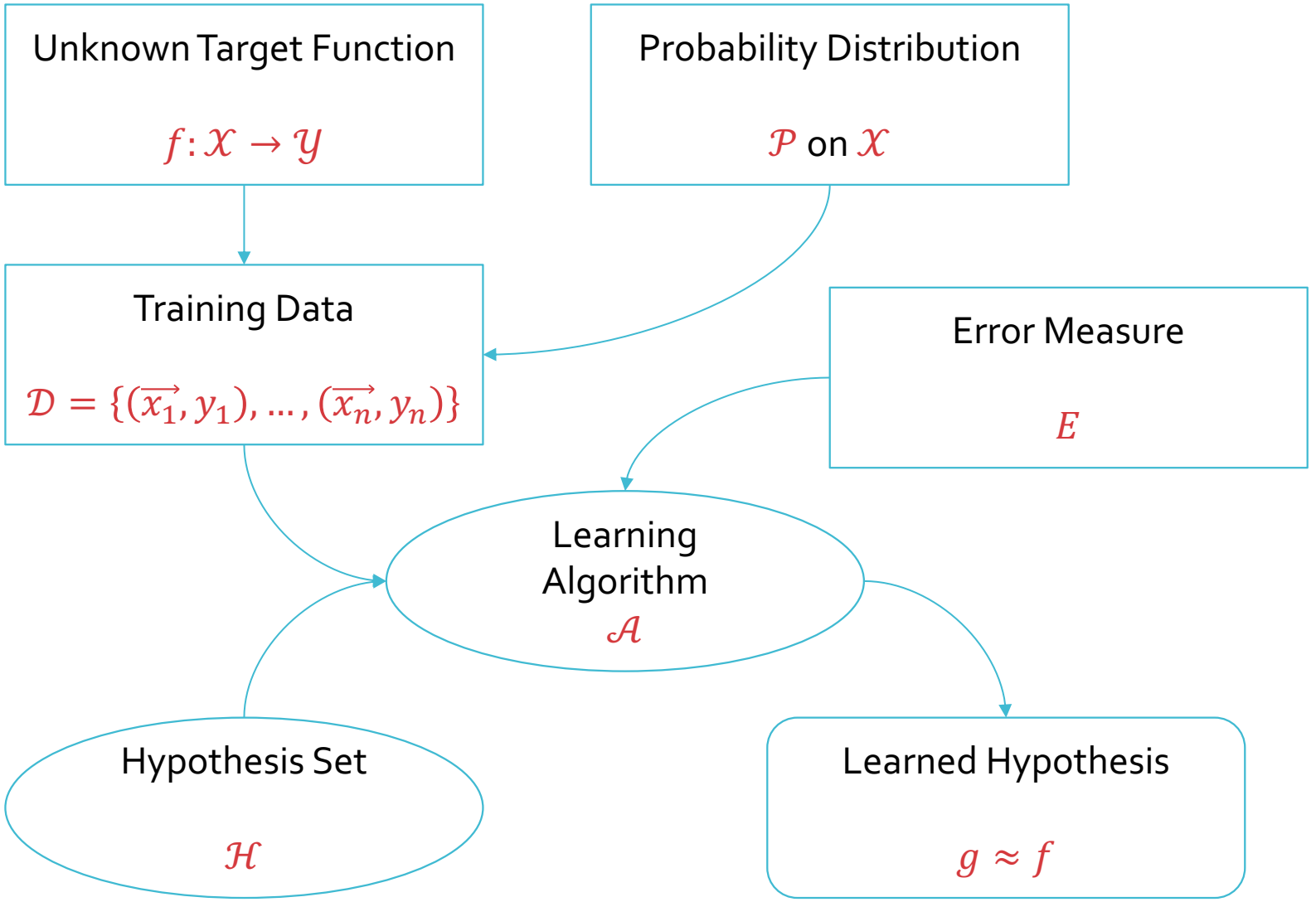
- For personalized coupons:

| | | $f(\vec{x})$ | |
|---|---|---|---|
| | | +1 | -1 |
| $h(\vec{x})$ | +1 | 0 | 1 |
| | -1 | 100 | 0 |

For unlocking phones:

| | | $f(\vec{x})$ | |
|---|---|---|---|
| | | +1 | -1 |
| $h(\vec{x})$ | +1 | 0 | 1000 |
| | -1 | 1 | 0 |

# Error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Training Data
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Error Measure
$E$
Learning Algorithm
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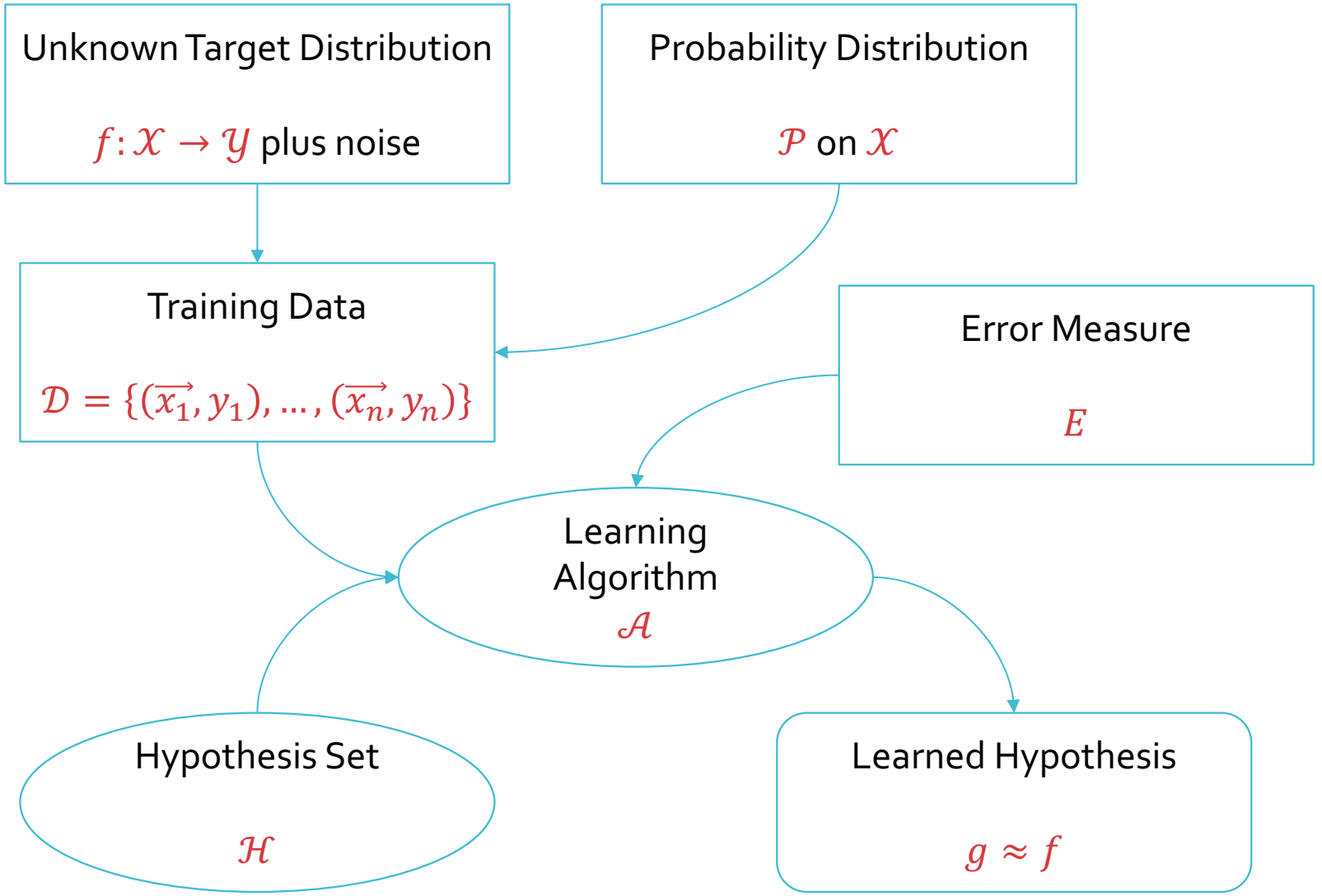
# Noise

- The target function is not always deterministic, it is sometimes **stochastic**
- Instead of a target function, a target **distribution**
- Instead of $y = f(\vec{x})$, $P\{y|\vec{x}\}$
  - $y = f(\vec{x}) + \epsilon$ where $\epsilon \sim N(0, \sigma^2)$

# Noise


Unknown Target Distribution
$f: \mathcal{X} \rightarrow \mathcal{Y}$ plus noise
Probability Distribution
$\mathcal{P}$ on $\mathcal{X}$
Training Data
$\mathcal{D} = \{(\overrightarrow{x_1}, y_1), \ldots, (\overrightarrow{x_n}, y_n)\}$
Error Measure
$E$
Learning Algorithm
$\mathcal{A}$
Hypothesis Set
$\mathcal{H}$
Learned Hypothesis
$g \approx f$